Computation of Steady Nozzle Flow by a Time-Dependent Method

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Theme

THE equations of motion governing steady, inviscid flow are of a mixed type, that is, hyperbolic in the supersonic region and elliptic in the subsonic region. These mathematical difficulties may be removed by using the so called time-dependent method, where the governing equations become hyperbolic everywhere. The steady-state solution may be obtained as the asymptotic solution for large time. This technique has been used to compute converging-diverging nozzle flows by Prozan (as reported by Saunders[1] and Cuffel et al.[2]), Migdal et al.,[3] Wehofer and Moger,[4] Laval,[5] and Serra.[6] This technique has also been used to compute converging nozzle flows by Wehofer and Moger[4] and Brown and Ozcan.[7] While the results of the preceding calculations are for the most part good, the computational times are rather large. In addition, although the computer program of Ref. 6 included a centerbody and those of Refs. 4 and 7 included the exhaust jet, none of the preceding codes is able to calculate both, that is, plug nozzles. Therefore, the object of this research was to develop a production type computer program capable of solving converging, converging-diverging, and plug two-dimensional nozzle flows in computational times of 1 min or less on a CDC 6600 computer.

Contents

The nonconservation form of the Euler equations for two-dimensional, inviscid, isentropic, rotational flow of a perfect gas are solved. The physical plane is mapped into a rectangular computational plane. The interior mesh points are computed using the MacCormack[8] scheme. The inlet, wall and centerbody, and exhaust jet boundary mesh points are calculated using a reference-plane characteristic scheme. The exit mesh points are computed using linear extrapolation for supersonic flow and a characteristic scheme for the subsonic case.

The results in the present study were obtained using a CDC 6600 computer. The computational times given are the central processor time not including compilation. In order to compare these results with those of other investigators, Table 1 is given (see backup paper for references).

The initial data in each case were computed internally by the program assuming one-dimensional, steady, isentropic flow with area change. When the relative change in axial velocity in the throat and downstream regions was less than a prescribed convergence tolerance, the flow was assumed to have reached steady state. The convergence tolerance was found to be a function of the mesh spacing, flow speed, and nozzle geometry. For the results presented here a convergence tolerance of 0.003% for flows without exhaust jet calculations and 0.005% for flows with exhaust jet calculations was employed.

Table 1 Relative machine speeds

Computer	Relative machine speed
IBM 7094, IBM 360/50	0.1
IBM 360/65	0.3
IBM 360/75, Univac 1108	0.5
CDC 6600	1.0

The present method was used to compute the steady-state solution for flow in a 45°–15° conical, converging-diverging nozzle (Fig. 1a). The Mach number contours and wall pressure ratio are shown in Fig. 2 and agree well with the experiments of Cuffel et al. The computed discharge coefficient is 0.983 as compared with the experimental value of 0.985. A 21 × 8 mesh was used, which required 301 solution planes and a time of 35 sec. This case was also solved by Prozan,[2] Migdal et al., Laval, and Serra. While the details of Prozan's computation were not reported by Cuffel et al., Saunders reported a time of

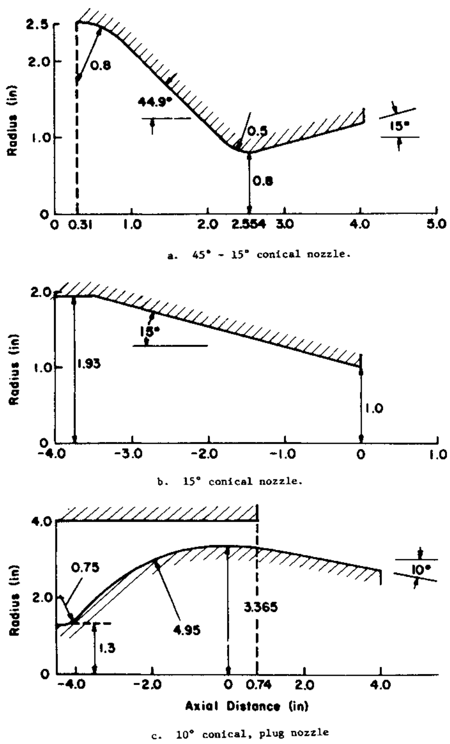

Fig. 1 Nozzle geometries.

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Index categories: Subsonic and Transonic Flow; Supersonic and Hypersonic Flow; Nozzle and Channel Flow.

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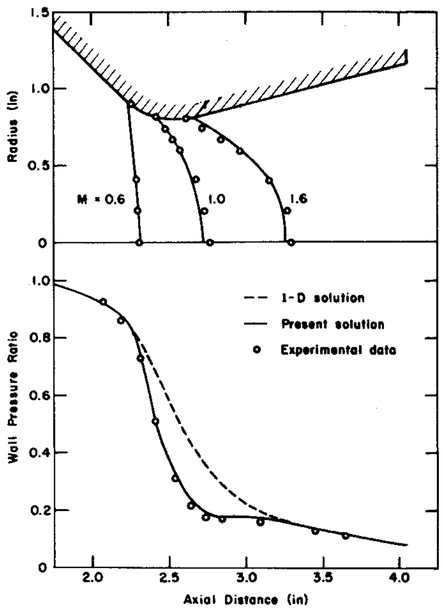

Fig. 2 Mach number contours (above) and wall pressure ratio for 45°–15° conical nozzle.

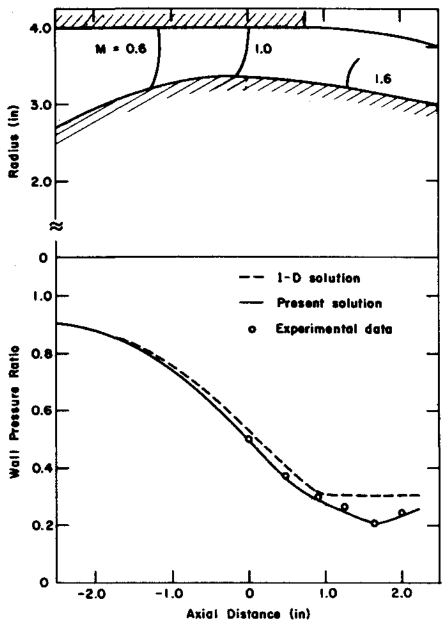

Fig. 4 Mach number contours (above) and plug pressure ratio for 10° conical, plug nozzle.

45 min on a CDC 3200 (23 × 11 mesh) for computing the flow in a nozzle with a large radius of curvature. Migdal et al. used less than 5 min on an IBM 360/75. Laval used on the order of 2 hr on an IBM 360/50 (61 × 21 mesh). Serra required 80 min on a Univac 1108 (3000 mesh points). In addition, a relaxation solution (Prozan and Kooker[9]) required 5 to 10 min on an IBM 7094 (21 × 11 mesh).

The present method was also used to compute the steady-state flow in a 15° conical, converging nozzle (Fig. 1b). The Mach number contours and wall pressure ratio for a nozzle pressure ratio of 2.0 are shown in Fig. 3 and agree well with the experiments of Thornock.[10] The computed discharge coefficient is 0.957 as compared with the experimental value of 0.960. A 23 × 7 mesh was used, which required 249 solution planes and a time of 29 sec. The numerical solution of Wehofer and Moger

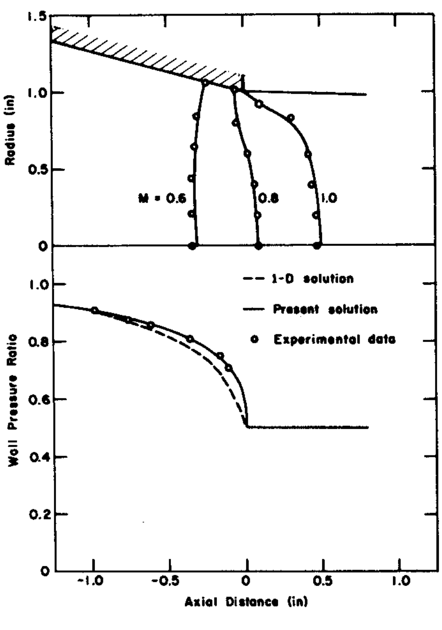

Fig. 3 Mach number contours (above) and wall pressure ratio for 15° conical nozzle.

required over 2 hours on an IBM 360/50 (47 × 11 mesh), while Brown and Ozcan's calculation required 17 minutes on an IBM 360/65 (20 × 6 mesh).

Finally, the present method was used to calculate the flow in a 10° conical, plug nozzle (Fig. 1c). The Mach number contours and plug pressure ratio for a nozzle pressure ratio of 3.29 are shown in Fig. 4 and agree well with the experiments of Bresnahan and Johns.[11] A 31 × 6 mesh was used, which required 327 solution planes and a time of 52 sec. The author is unaware of any other time-dependent analysis of plug nozzles.

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